Intermediate representations

Advanced Compiler Construction Michel Schinz – 2021-03-04

The term intermediate representation (IR) or intermediate language designates the data-structure(s) used by the compiler to represent the program being compiled.

Choosing a good IR is crucial, as many analyses and transformations (e.g. optimizations) are substantially easier to perform on some IRs than on others. Most non-trivial compilers actually use several IRs during the compilation process, and they tend to become more low-level as the code approaches its final form.

Intermediate representations

divisor (GCD) of 2016 and 714. The L₃ version of that fragment could be: (**rec** loop ((x 2016) (y 714)) (if (= 0 y))(int-print x) (loop y (% x y))))



To illustrate the differences between the various intermediate representations, we will use a program fragment to compute and print the greatest common

IR#1: CPS/L3

- A **functional IR** is an intermediate representation that is close to a (very) simple functional programming language. Typical functional IRs have the following characteristics:
 - all primitive operations (e.g. arithmetic operations) are performed on atomic values (variables or constants), and the result of these operations is always named,
 - variables cannot be re-assigned.
- As we will see later, some of these characteristics are shared with more mainstream IRs, like SSA.
- CPS/L₃ is the functional IR used by the L₃ compiler.

CPS/L₃: a functional IR

Local continuations

A crucial notion in CPS/L₃ is that of **local continuation**. A local continuation is similar to a (local) function but with the following restrictions:

(described later),

- continuations never return, and must therefore be invoked in tail position. These restrictions enable continuations to be compiled much more efficiently than normal functions. This is the only reason why continuations exist as a separate construct.

- continuations are not "first class citizens": they cannot be stored in variables or passed as arguments – the only exception being the return continuation

Uses of continuations

Continuations are used for two purposes in CPS/L₃:

- 1. To represent code blocks which can be "jumped to" from several locations, by invoking the continuation.
- 2. To represent the code to execute after a function call. For that purpose, every function gets a continuation as argument, which it must invoke with its return value.

$$T ::= (let_{p} ((N (PA ...) | (let_{c} ((N (cnt (I) | (let_{f} ((N (fun (I) | (app_{c} AA ...)) | (app_{f} ANA ...)) | (if (CAA) NN) | (if (CAA) NN) | (halt A)$$
$$N ::= name$$
$$L ::= literal (e.g. integer, ch A ::= N | L$$
$$P ::= + | - | * | / |% | ...$$
$$C ::= < | <= | = | ...$$

CPS/L₃ grammar

.))) T) N...) T))) T) NN...) T)) ...) T)

haracter, boolean or unit)

CPS/L₃ local binding

(let_p ((n (p a₁...))) e) Binds the name n to the result of the a₁, ... in expression e. The primitive p cannot be a logical (are only meant to be used in condit

Binds the name n to the result of the application of primitive p to the value of

The primitive p cannot be a logical (i.e. boolean) primitive, as such primitives are only meant to be used in conditional expressions – see later.

 $(let_f ((f_1 (f_{1} (f_{1} (r_1 n_{1,1}...) b_1)) ...) e))$ Binds the names f_1, \ldots to functions with arguments $n_{1,1}, \ldots$ and return continuation r_1, \ldots in expression e. The functions can be mutually recursive. is interpreted as returning from the function. (**app**_f f r a₁...)

return continuation of the current function.

CPS/L₃ functions

- The return continuation takes a single argument: the return value. Applying it
- Applies the function bound to f to return continuation r and arguments a_1, \ldots The name r must either be bound by an enclosing let_c or be the name of the

- $(let_{c} ((c_{1} (cnt (n_{1,1}...) b_{1})) ...) e)$ Binds the names c_1, \ldots to local continuations with arguments $n_{1,1}, \ldots$ and body b_1, \ldots in expression e.
- Interpretation: similar to a local function that does not return. (**app**_c c a₁...)
- Applies the continuation bound to c to the value of a_1, \ldots The name c must either be bound by an enclosing let_c or be the name of the return continuation of the current function. Interpretation:
 - function, with the given return value.
 - if c designates a local continuation, jump to it (with arguments), - if c designates the current return continuation, return from the current

CPS/L₃ local continuations

CPS/L₃ control constructs

(**if** (p a₁ a₂) c_t c_f)

continuations.

The primitive p must be a logical primitive. continuations. It is therefore a conditional version of app_c .

(halt a)

Halts program execution, exiting with the value of a, which must be an integer.

- Tests whether the condition p is true for the value of a_1 and a_2 , then applies continuation c_t if it is, or c_f if it isn't. Both c_t and c_f must be parameterless
- Note: if is a branching form of continuation application for parameterless

Continuation scopes

For example, in the following code: $(let_c ((c_0 (cnt (r) (app_f print r))))$ $(let_f ((f (fun (c_1 x)))))$ $(let_p ((t (+ x x))))$ (c₁ t)))))) **c**₀ is *not* visible in the body of **f**! them, and can therefore be compiled efficiently.

- The scoping rules of CPS/L₃ are mostly the "obvious ones". The only exception is the rule for continuation variables, which are not visible in nested functions!

This guarantees that continuations are truly local to the function that defines

CPS/L₃ syntactic sugar

To make CPS/L₃ programs easier to read and write, we allow the collapsing of nested occurrences of let_p and let_c expressions to a single let * expression. We also allow the elision of app_f and app_c in applications. Example:

> $(let_p ((c_1 (+ 1 2)))$ (let_p ((c₂ (+ 2 3)))

 $(let_p ((c_3 (+ c_1 c_2))))$... $(app_f f c_3)))$

 $\mathbf{\hat{l}}$

1 2)) 2 3)) $C_1 C_2)))$

GCD in CPS/L₃

```
The CPS/L<sub>3</sub> version of the GCD program fragment looks as follows:
 (let<sub>c</sub> ((loop
            (cnt (x y))
               (let* ((ct (cnt ()
                                 (app<sub>f</sub> print x)))
                        (cf (cnt ()
                                 (let<sub>p</sub> ((t (% x y)))
                                   (app_c loop y t))))
                  (if (= y 0) ct cf))))
       (app<sub>c</sub> loop 2016 714))
(To simplify, no return continuation is passed to print).
```

Translation of CL₃ to CPS/L₃

The translation from CL₃ to CPS/L₃ is specified as a function denoted by $\llbracket \cdot \rrbracket$ and taking two arguments:

- 1. T, the CL₃ term to be translated,
- This function is written in a "mixfix" notation, as follows:
- and must return a CPS/L₃ term that includes both:
 - 1. the translation of the CL₃ term T, and
 - (the translation of) T.

Translating CL₃ to CPS/L₃

2. C, the translation context, a CPS/L_3 term containing a hole into which an atom containing the value of the translated term has to be plugged.

2. the context C with its hole plugged by an atom containing the value of

Translation context

the CL₃ expression surrounding the one being translated. unknown atom.

knows the atom in question.

- The translation context is a partial CPS/L₃ term representing the translation of
- It is partial in the sense that it contains a single **hole**, representing an as-yet-
- This hole is meant to be plugged eventually by the translation function, once it

Translation context

represented as a meta-function with a single argument – the hole. node where the argument of halt is not yet known: λx (halt x)

called C, its hole can be plugged with the atom 1 as follows: C[1]

producing the complete CPS/L_3 term (halt 1).

- In what follows, and in the implementation, the translation context will be For example, the following meta-function represents a partial CPS/L₃ halt
- A context being a meta-function, its hole can be plugged simply by function application at the meta-level. For exemple, assuming the above context is

The translation in Scala

In Scala – the meta-language in the L₃ project – the translation function $\llbracket \cdot \rrbracket$ is defined as a function with the following profile: def CL3ToCPS(t: CL3Tree, c: Atom ⇒ CPSTree): CPSTree In the body of that function, plugging the context c with an atom bound to a Scala value named a is done using Scala function application: c(a)

CL₃ to CPS/L₃ translation (1)

Note: in the following expressions, all underlined names are fresh. [a] C where a is an atom (i.e. name or literal) = C[a] $[(let ((n_1 e_1) (n_2 e_2) ...) e)]C = id = identity primitive$

 $[(let ((n_2 e_2)...)e)]C))$

[(let ()e)]C =[e] C

- $[e_1](\lambda v_1 (let_p ((n_1 (id v_1)))) (returns its argument))$

CL₃ to CPS/L₃ translation (2)

CL₃ to CPS/L₃ translation (3)

 $[(if (@p e_1 ...) e_2 e_3)] C where p is a logical primitive = (let_c ((c (cnt (r) C[r]))) (let_c ((ct (cnt () [[e_2]](\lambda v_2 (app_c c v_2))))) (let_c ((cf (cnt () [[e_3]](\lambda v_3 (app_c c v_3))))) [[e_1]](\lambda v_1 ... (if (@p v_1 ...) ct cf)))))) [(if e_1 e_2 e_3)] C = [(if (@= e_1 #f) e_3 e_2)] C$

CL₃ to CPS/L₃ translation (4)

[(@p e₁ e₂...)]C where p is a logical primitive =
 [(if (@p e₁ e₂...) #t #f)]C
[(@p e₁ e₂...)]C where p is not a logical primitive =
 left as an exercise
[(halte)]C =
 left as an exercise

In which context should a complete program be translated? (signalling no error), that is:

 λv (halt 0)

whole program, e.g. use it as the exit code instead of 0, print it, etc.



The simplest answer is a context that halts execution with an exit code of 0

An alternative would be to do something with the value v produced by the



Translate the following L₃ expression: (f 1 2) in the initial context, λv (halt 0).

Exercise

Better translation of CL₃ to CPS/L₃

Improving the translation

The translation presented before has two shortcomings: 1. it produces terms containing useless continuations, and 2. it produces suboptimal CPS/L₃ code for some conditionals. One solution to improve the translation is to define several different to translate appears.

- translations depending on the source (i.e. L₃) context in which the expression

Useless continuations

The first problem can be illustrated with the CL₃ term: (letrec ((f (fun (g) (g)))) f) which – in the empty context – gets translated to: (let_{f}) ((f (fun (c g)) (appf g j))))

f) instead of the equivalent and more compact: $(let_f ((f (fun (c g) (app_f g c))))$ f)

 $(let_c ((j (cnt (r) (app_c c r))))$

Suboptimal conditionals (1)

The second problem can be illustrated with the CL₃ term: (if (if a b #f) x y) which, in the empty context, gets translated to: (let* ((ci1 (cnt (v1) v1)) (ct1 (cnt () (app_c ci1 x))) (cf1 (cnt () (app_c ci1 y))) (ci2 (cnt (v2) (if (= v2 #f) cf1 ct1))) $(ct2 (cnt () (app_c ci2 b)))$ (cf2 (cnt () (app_c ci2 #f)))) (if (= a #f) cf2 ct2))

Suboptimal conditionals (2)

A much better translation for: (if (if a b #f) x y) would be:

(let* ((ci1 (cnt (v1) v1)) (ct1 (cnt () (app_c ci1 x))) (cf1 (cnt () (app_c ci1 y))) (cal (cnt () (if (= b #f) cfl ctl))) (if (= a #f) cf1 ca1))

which immediately applies continuation cf1 if a is false.

Source contexts

better if it depended on the source context in which the expression to translate appears.

- In the first example, the function call could be translated more efficiently because it appears as the last expression of the function (i.e. it is in tail position).
- For the second example, the nested if expression could be translated more efficiently because it appears in the condition of another if expression and one of its branches is a simple boolean literal (here #f). Therefore, instead of having one translation function, we should have several: one per source context worth considering!

These two problems have in common the fact that the translation could be

separate ones:

- must be plugged with an atom containing the value of the term. continuation c. This continuation is to be applied to the value of the term. continuations, c_t and c_f . The continuation c_t is to be applied when the term evaluates to a true value, while the continuation c_f is to be applied when it
- 1. $\llbracket \cdot \rrbracket_N C$, taking as before a term to translate and a context C, whose hole 2. $\llbracket \cdot \rrbracket_T$ c, taking a term to translate and the name of a one-parameter 3. $\llbracket \cdot \rrbracket_C c_t c_f$, taking a term to translate and the names of two parameterless
- evaluates to a false value.

A better translation

To solve the two problems, we split the single translation function into three

The non-tail translation

 $\llbracket \cdot \rrbracket_N$ is called the **non-tail** translation as it is used in non-tail contexts. That is, when the work that has to be done once the term is evaluated is more complex than simply applying a continuation to the term's value.

The non-tail translation

 $[[@p e_1 e_2 ...)]_N C$ where p is not a logical primitive = $[e_1]_N(\lambda v_1 [e_2]_N(\lambda v_2...)$ $(let_p ((n (p v_1 v_2...))))$ C[n]))

For example, the arguments of a primitive are always in a non-tail context, since once they are evaluated, the primitive has to be applied on their value:

The **tail** translation $\llbracket \cdot \rrbracket_T$ is used whenever the context passed to the simple translation has the form λv (app_c c v). It gets as argument the name of the continuation c to which the value of expression should be applied.

The tail translation

For example, the previous translation of function definition: $[[(letrec ((f_1 (fun (n_{1,1} n_{1,2}...) e_1))...) e)]]C =$ $(let_f ((f_1 (f_1 (f_1 n_{1,1} n_{1,2} ...))))$ $[e_1](\lambda v (app_c c v))) ...)$ [e]C) becomes: $[(letrec ((f_1 (fun (n_{1,1} n_{1,2}...) e_1))...) e)]_N C =$ (let_f ((f₁ (fun (\underline{c} n_{1,1} n_{1,2}...) [[e₁]]_T c))...)

 $[e]_{N}C)$

The tail translation

The cond translation

The **cond** translation $[\cdot]_C$ is used whenever the term to translate is a condition to be tested to decide how execution must proceed. It gets two continuations names as arguments: the first is to be applied when the condition is true, while the second is to be applied when it is false.

The cond translation

This translation is used to handle the condition of an if expression: $\begin{bmatrix} (if e_1 e_2 e_3) \end{bmatrix}_N C = \\ (let_c ((c (cnt (r) C[r]))) \\ (let_c ((ct (cnt () [[e_2]_T c]))) \\ (let_c ((cf (cnt () [[e_3]_T c]))) \\ [[e_1]_C ct cf)))) \end{bmatrix}$

The cond translation

The better translation in Scala

functions, with the following profiles: **def** nonTail(t: CL3Tree) (c: Atom ⇒ CPSTree): CPSTree def tail(t: CL3Tree, c: Symbol): CPSTree def cond(t: CL3Tree, ct: Symbol, cf: Symbol): CPSTree

- In the compiler, the three translations are simply three mutually-recursive

IR #2: standard RTL/CFG

Register-transfer language

variables) and store the result in another virtual register. could be written $x \leftarrow y + z$.

(+ here).

number of virtual registers is usually not bounded.

- A register-transfer language (RTL) is a kind of intermediate representation in which most operations compute a function of several virtual registers (i.e.
- For example, the instruction adding variables y and z, storing the result in x
- Such instructions are sometimes called **quadruples**, because they typically have four components: the three variables (x, y and z here) and the operation
- RTLs are very close to assembly languages, the main difference being that the

A control-flow graph (CFG) is a directed graph whose nodes are the individual instructions of a function, and whose edges represent control-flow. More precisely, there is an edge in the CFG from a node n_1 to a node n_2 if and only if the instruction of n₂ can be executed immediately after the instruction of n_1 .

Control-flow graph

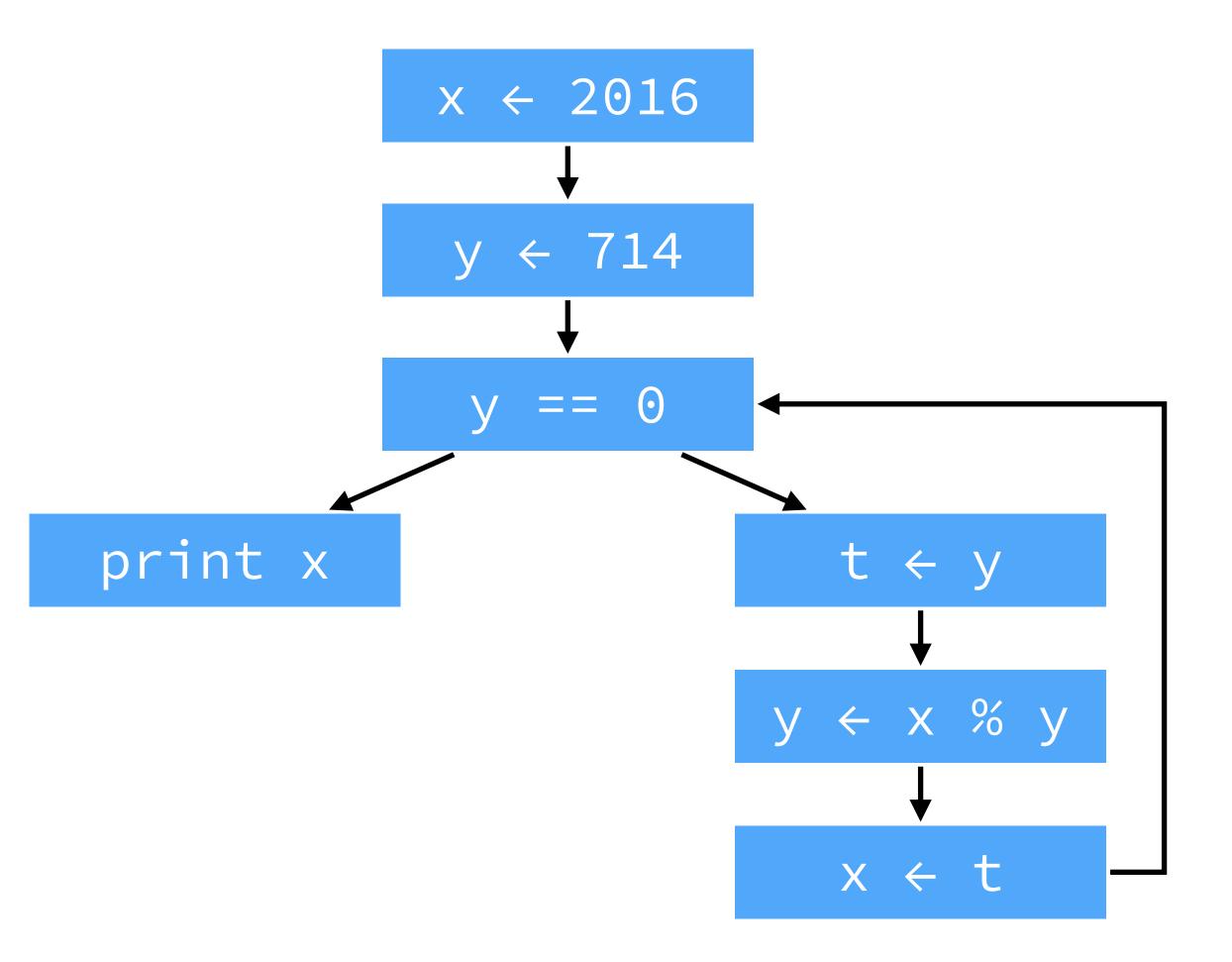
RTL/CFG is the name given to intermediate representations where each contain RTL instructions.

especially those for imperative languages.



- function of the program is represented as a control-flow graph whose node
- This kind of representation is very common in the later stages of compilers,

Computation of the GCD of 2016 and 714 in a typical RTL/CFG representation.

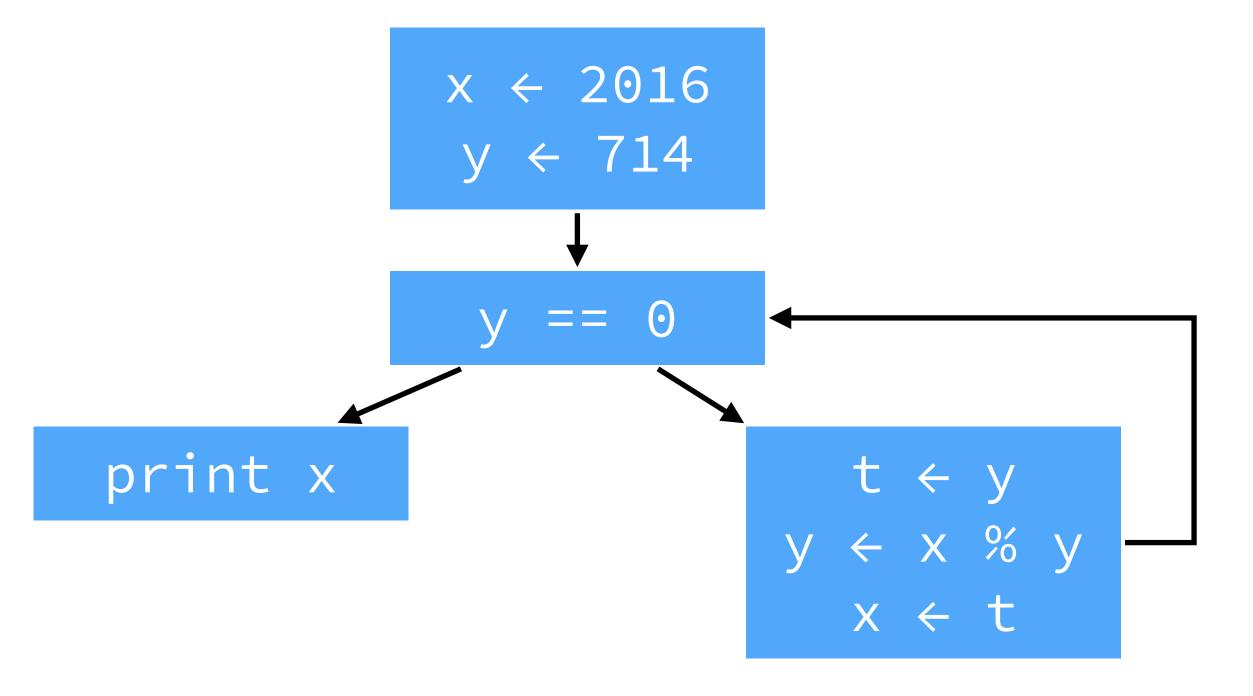




Basic blocks

A **basic block** is a maximal sequence of instruction for which control can only enter through the first instruction of the block and leave through the last. Basic blocks are sometimes used as the nodes of the CFG, instead of individual instructions. This has the advantage of reducing the number of nodes in the CFG, but also complicates data-flow analyses.

instructions.





The same examples as before, but with basic blocks instead of individual

RTL/CFG issues

This is because a single variable can be assigned multiple times. Is it possible to improve RTL/CFG so that these optimizations can be performed without prior analysis? Yes, by using a single-assignment variant of RTL/CFG!

- One problem of RTL/CFG is that even very simple optimizations (e.g. constant) propagation, common-subexpression elimination) require data-flow analyses.

IR #3: RTL/CFG in SSA form

each variable has only one definition in the program. if it is inside a loop – hence the qualifier static. we will see.

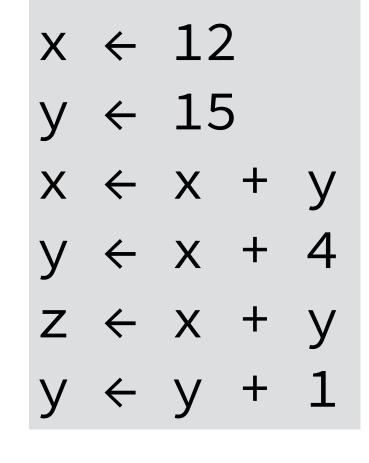
be transformed so that they are.



An RTL/CFG program is said to be in **static single-assignment** (**SSA**) form if

- That single definition can be executed many times when the program is run –
- SSA form is popular because it simplifies several optimizations and analysis, as
- Most (imperative) programs are not naturally in SSA form, and must therefore

Straight-line code





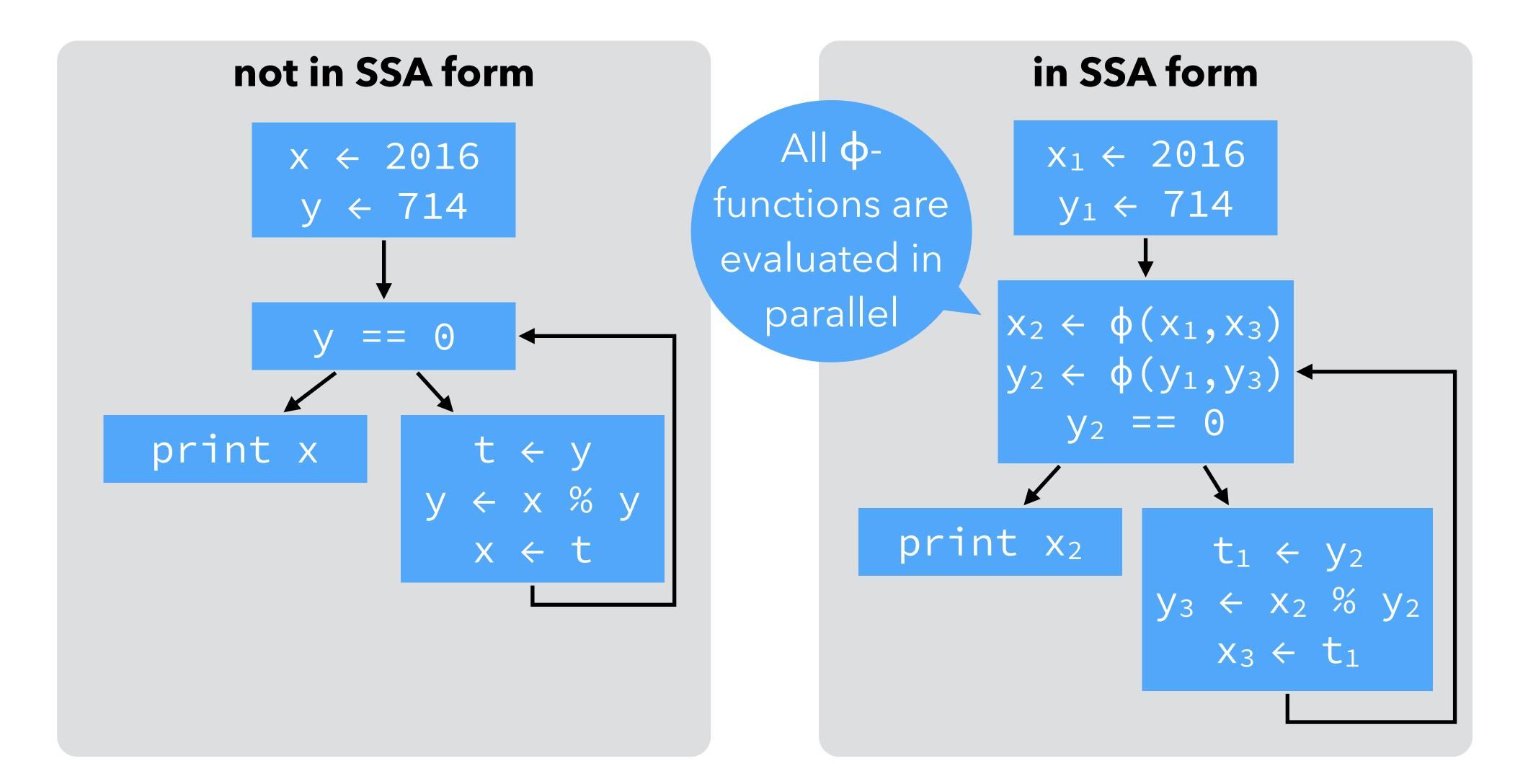
$$x_{1} \leftarrow 12$$

 $y_{1} \leftarrow 15$
 $x_{2} \leftarrow x_{1} + y_{1}$
 $y_{2} \leftarrow x_{2} + 4$
 $z_{1} \leftarrow x_{2} + y_{2}$
 $y_{3} \leftarrow y_{2} + 1$

φ-functions

Join-points in the CFG – nodes with more than one predecessor – are more problematic, as each predecessor can bring its own version of a given name. To reconcile those different versions, a fictional **\phi-function** is introduced at the join point. That function takes as argument all the versions of the variable to reconcile, and automatically selects the right one depending on the flow of control.





φ-functions example

Evaluation of ϕ -functions

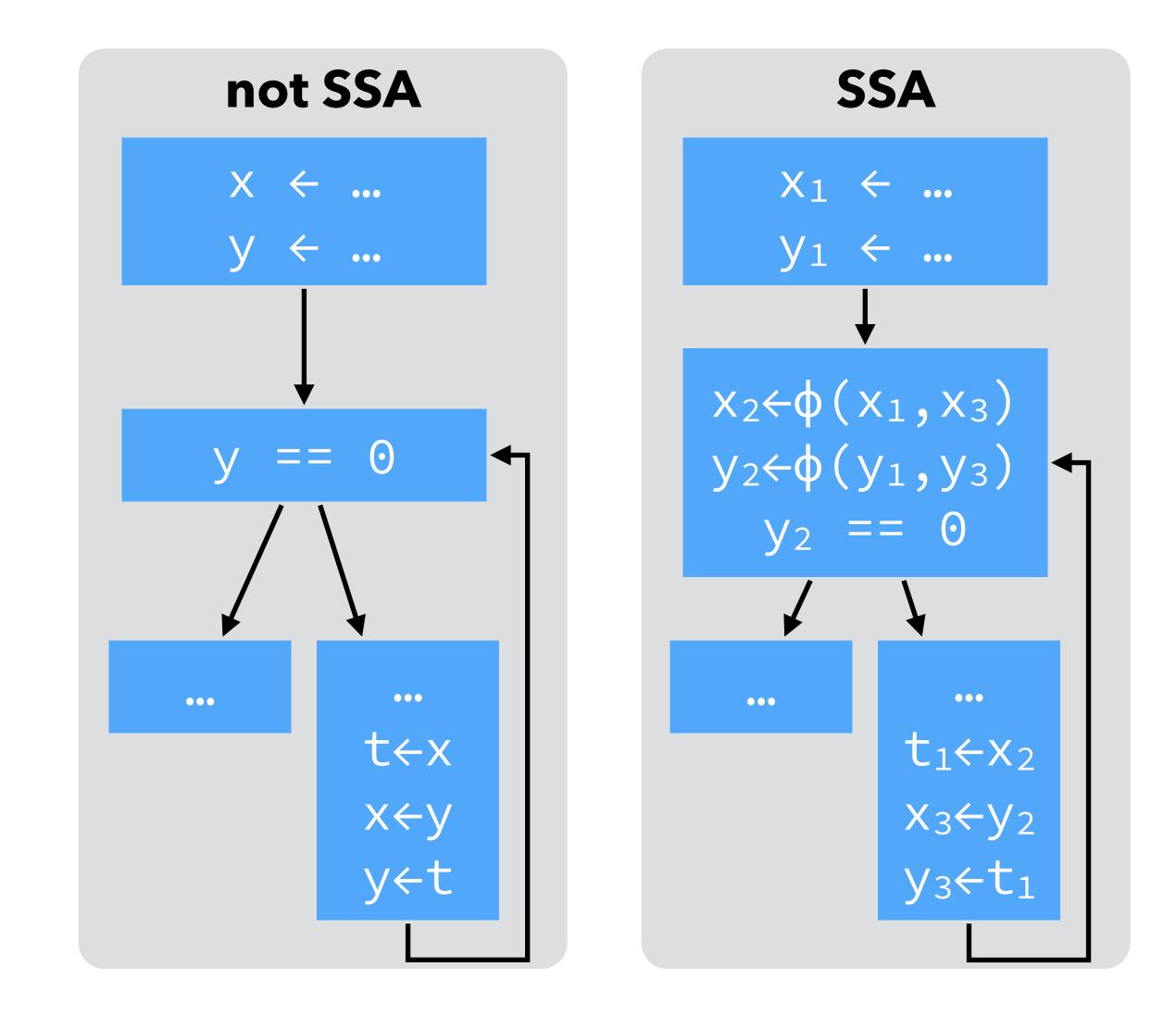
It is crucial to understand that all **φ**-functions of a block are evaluated *in parallel*, and not in sequence as the representation might suggest! To make this clear, some authors write **φ**-functions in matrix form, with one row per predecessor:

$$(x_2, y_2) \leftarrow \varphi \begin{pmatrix} x_1 & y_1 \\ x_3 & y_3 \end{pmatrix}$$

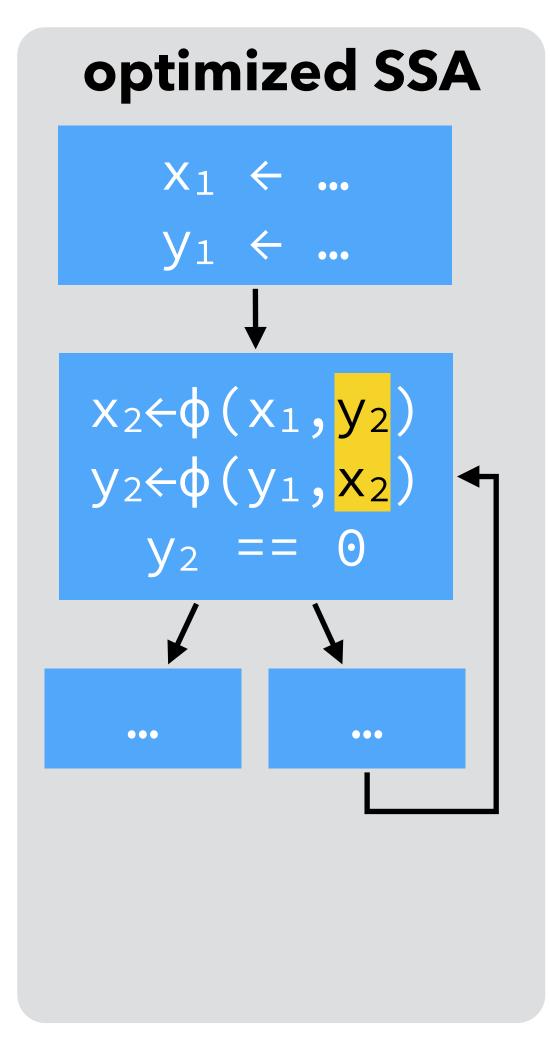
In the following slides, we will usually stick to the common, linear representation, but keep the parallel nature of ϕ -functions in mind.

instead of
$$\begin{array}{c} x_2 \leftarrow \phi(x_1, x_3) \\ y_2 \leftarrow \phi(y_1, y_3) \end{array}$$

Evaluation of ϕ -functions

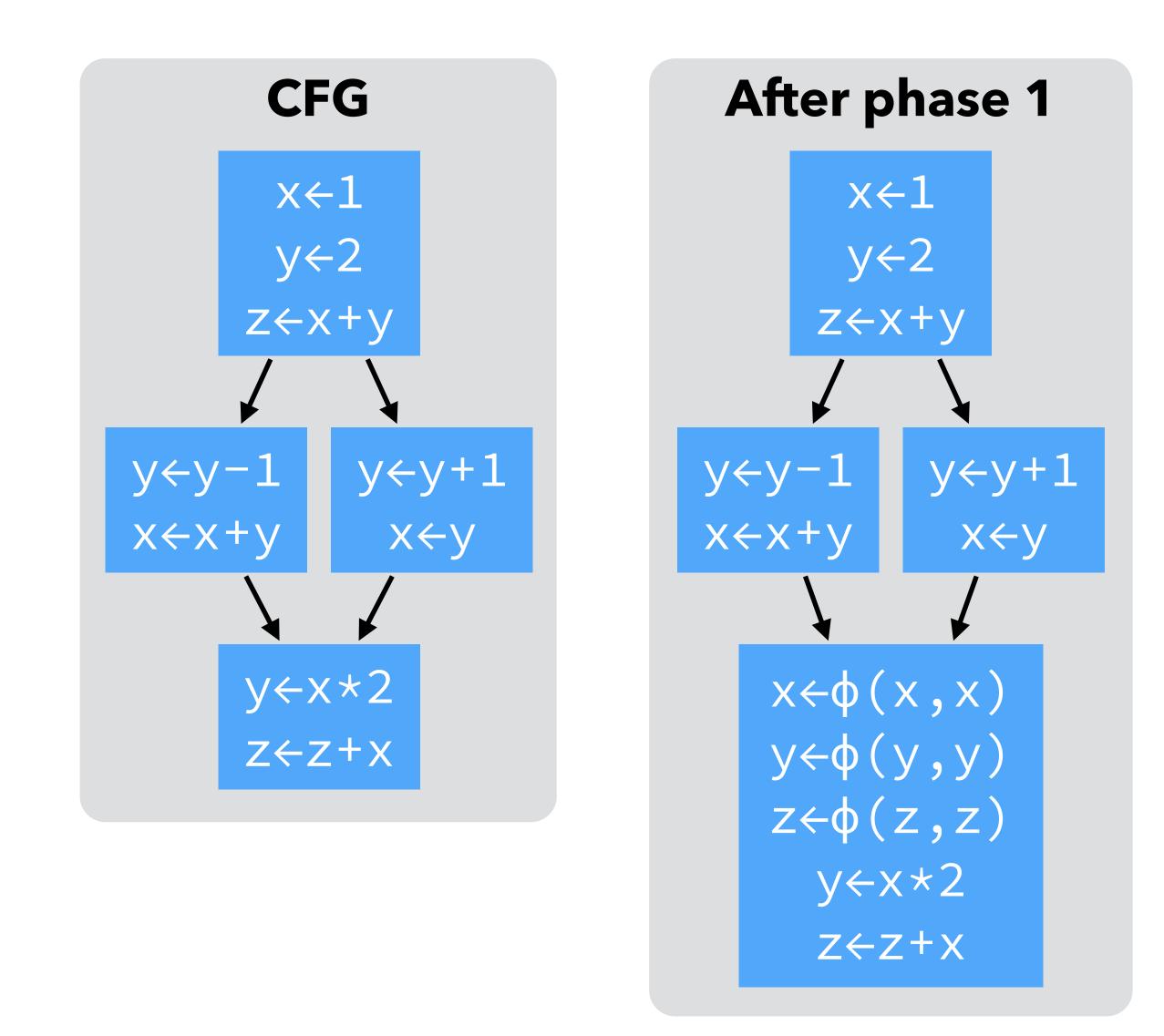


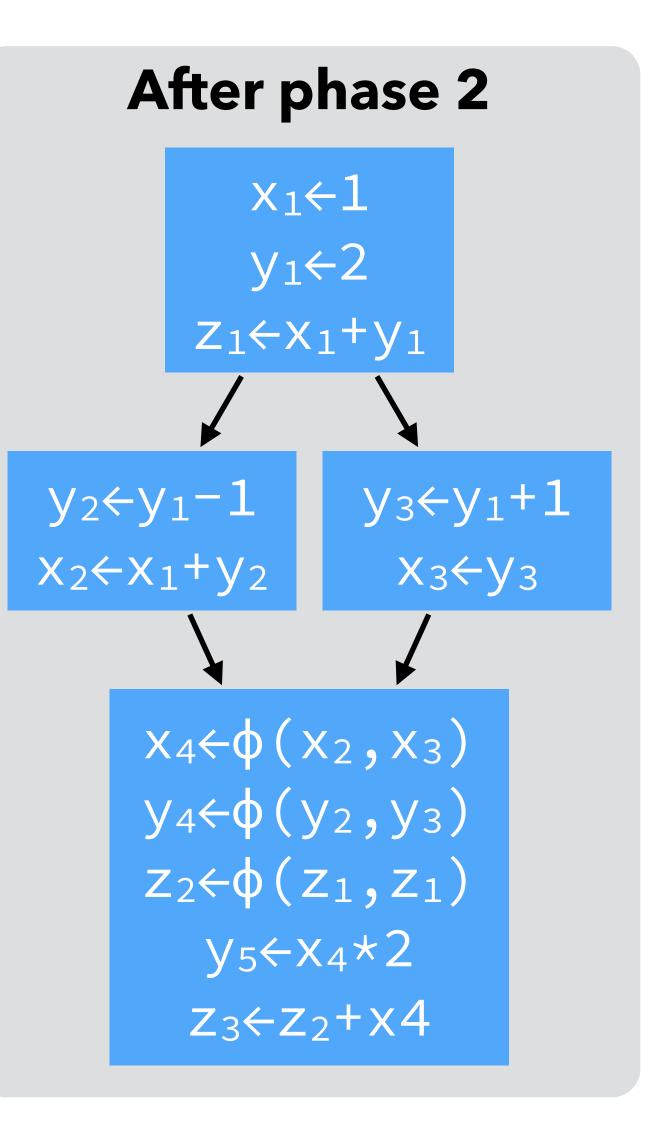
The following loop illustrates why ϕ -functions must be evaluated in parallel.

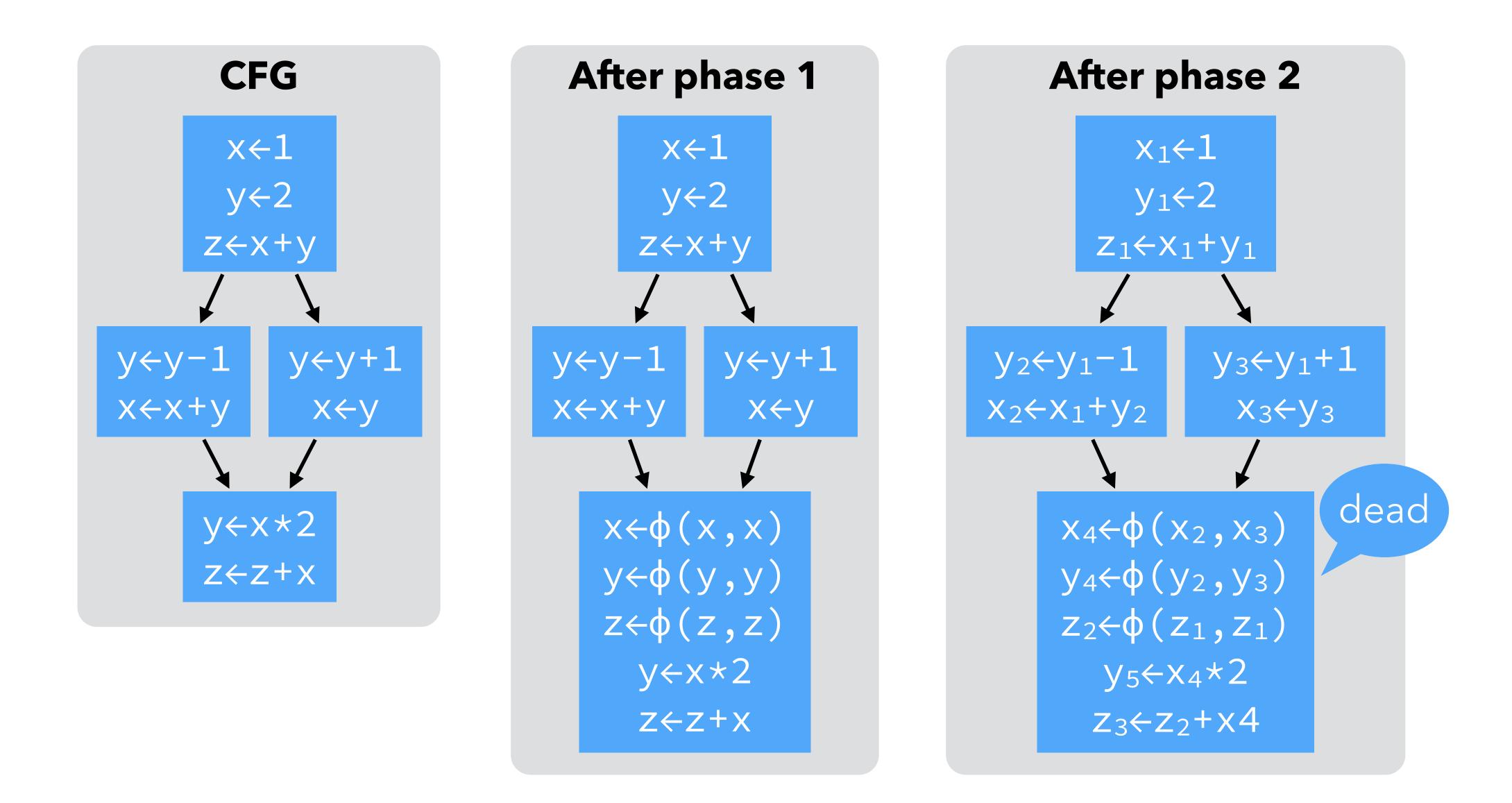


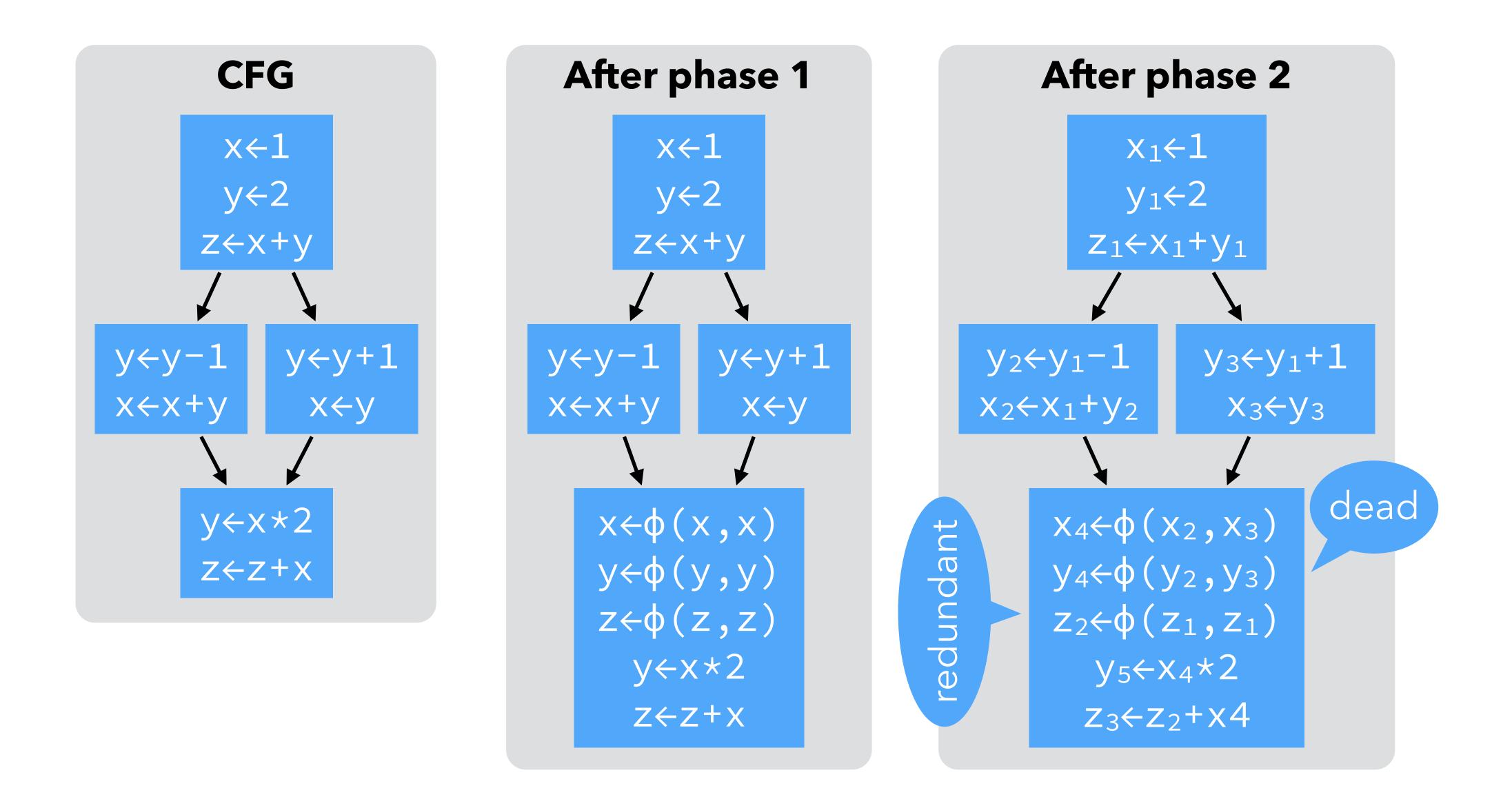
Naïve technique to build SSA form:

- for each variable x of the CFG, at each join point n, insert a ϕ -function of the form $x = \phi(x, ..., x)$ with as many parameters as n has predecessors, - compute reaching definitions, and use that information to rename any use of a variable according to the – now unique – definition reaching it.









Better building techniques

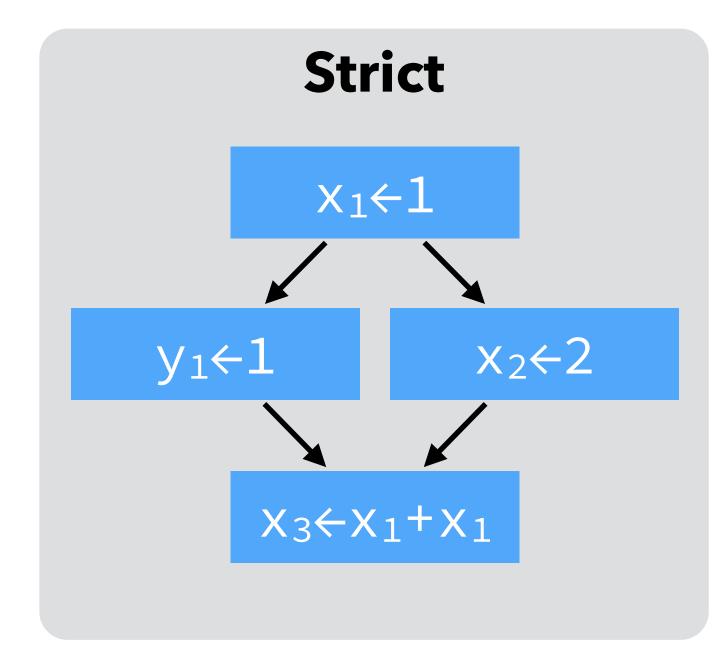
The naïve technique just presented works, in the sense that the resulting program is in SSA form and is equivalent to the original one. to be useful in practice. It builds the **maximal** SSA form. Better techniques exist to translate a program to SSA form.

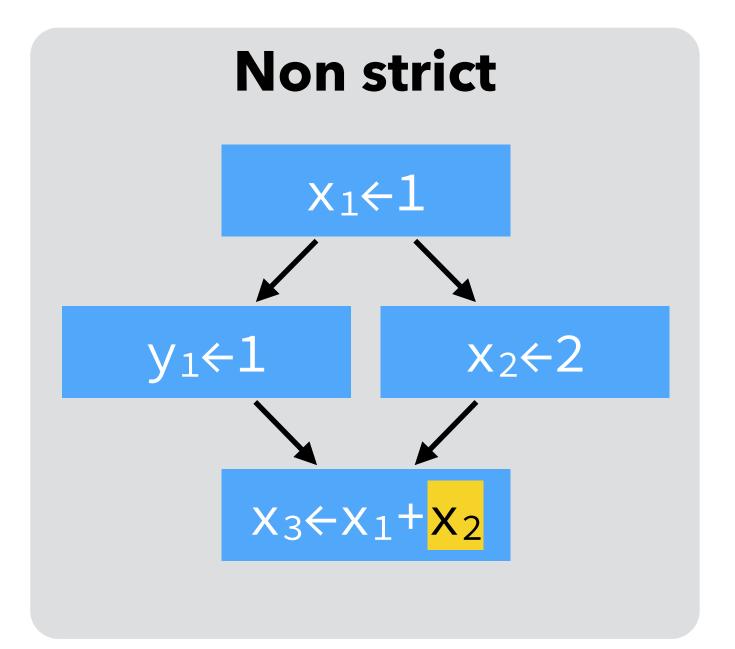
- However, it introduces too many ϕ -functions some dead, some redundant

Strict SSA form

A program is said to be in **strict SSA form** if it is in SSA form and all uses of a variable are dominated by the definition of that variable. (In a CFG, a node n_1 **dominates** a node n_2 if all paths from the entry node to n_2 go through n_1 .) Strict SSA form guarantees that no variable is used before being defined.

Strict SSA form







Comparing IRs

CPS/L₃ vs RTL/CFG in SSA

RTL/CFG in SSA form.

$RTL/CFG in SSA \cong CPS/L_3$

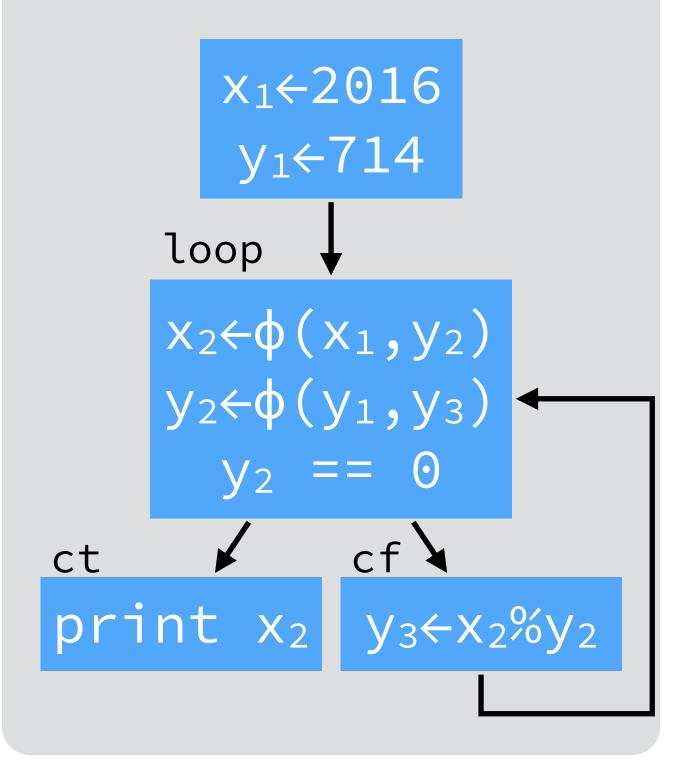
- (named) basic block ≅ continuation
 - ϕ -function \cong continuation argument
 - jump ≅ continuation invocation
 - strict form ≅ scoping rules

As the correspondences in the table below illustrate, CPS/L₃ is very close to



CPS/L₃ vs RTL/CFG in SSA

RTL/CFG in SSA form



CPS/L₃

Summary and references

Claim: continuation-based, functional IRs like CPS/L₃ are SSA done right, and should replace it – or, at the very least, ϕ -functions should be replaced by continuation arguments.

(This is fortunately starting to happen, e.g. the Swift Intermediate Language has basic-blocks with arguments.)

CPS/L₃ is heavily based on the intermediate representation presented by Andrew Kennedy in *Compiling with Continuations, Continued*, in Proceedings of the International Conference on Functional Programming (ICFP) 2007.

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