

Register allocation

Advanced Compiler Construction
Michel Schinz – 2026-04-02

Register allocation

Register allocation consists in:

- rewriting a program that makes use of an unbounded number of virtual or pseudo-registers,
- into one that only uses physical (machine) registers.

Some virtual registers might have to be **spilled** to memory.

Register allocation is done:

- very late in the compilation process – typically only instruction scheduling comes later,
- on an IR very close to machine code.

Setting the scene

We will do register allocation on an RTL with:

- n machine registers $R_0, \dots, R_{n-1}$ (some with non-numerical indexes like the link register R_{LK}),
- unbounded number of virtual registers $v_0, v_1, \dots$

Of course, virtual registers are only available before register allocation.

Running example

Euclid's algorithm to compute greatest common divisor.

In L₃

```
(defrec gcd  
  (fun (a b)  
    (if (= 0 b)  
      a  
      (gcd b (% a b)))))
```

In RTL

```
gcd:  R3 ← done  
      if R2 = 0 goto R3  
      R3 ← R2  
      R2 ← R1 % R2  
      R1 ← R3  
      R3 ← gcd  
      goto R3  
done: goto RLK
```

Calling conventions:

- the arguments are passed in R₁, R₂, ...
- the return address is passed in R_{LK},
- the return value is passed in R₁.

Register allocation example

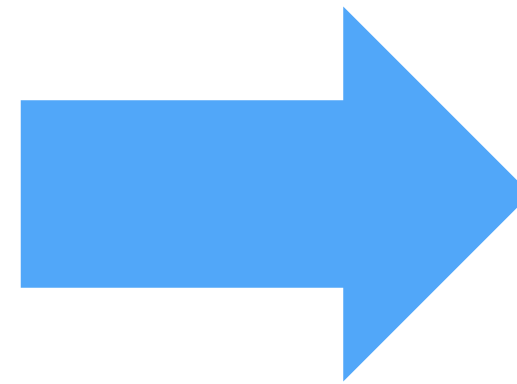
Before register allocation

```
gcd:  v0 ← RLK
      v1 ← R1
      v2 ← R2
loop: v3 ← done
      if v2 = 0 goto v3
      v4 ← v2
      v2 ← v1 % v2
      v1 ← v4
      v5 ← loop
      goto v5
done: R1 ← v1
      goto v0
```

R₁, R₂: parameters

R_{LK}: return address

allocable
registers:
R₁, R₂, R₃,
R_{LK}



After register allocation

```
gcd:
loop: R3 ← done
      if R2 = 0 goto R3
      R3 ← R2
      R2 ← R1 % R2
      R1 ← R3
      R3 ← loop
      goto R3
done: goto RLK
```

Allocation:

v₀ → R_{LK}

v₁ → R₁

v₂ → R₂

v₃, v₄, v₅ → R₃

Techniques

We will study two commonly used techniques:

1. register allocation by **graph coloring**, which:

- produces good results,
- is relatively slow,
- is therefore used mostly in batch compilers,

2. **linear scan** register allocation, which:

- produces average results,
- is very fast,
- is therefore used mostly in JIT compilers.

Both are **global**: they allocate registers for a whole function at a time.

Technique #1: graph coloring

Allocation by graph coloring

Register allocation can be reduced to graph coloring:

1. build the **interference graph**, which has:
 - one node per register – real or virtual,
 - one edge between each pair of nodes whose registers are live at the same time.
2. color the interference graph with at most K colors (K = number of available registers), so that all nodes have a different color than all their neighbors.

Problems:

- coloring is NP-complete for arbitrary graphs,
- a K -coloring might not even exist.

Interference graph example

Program

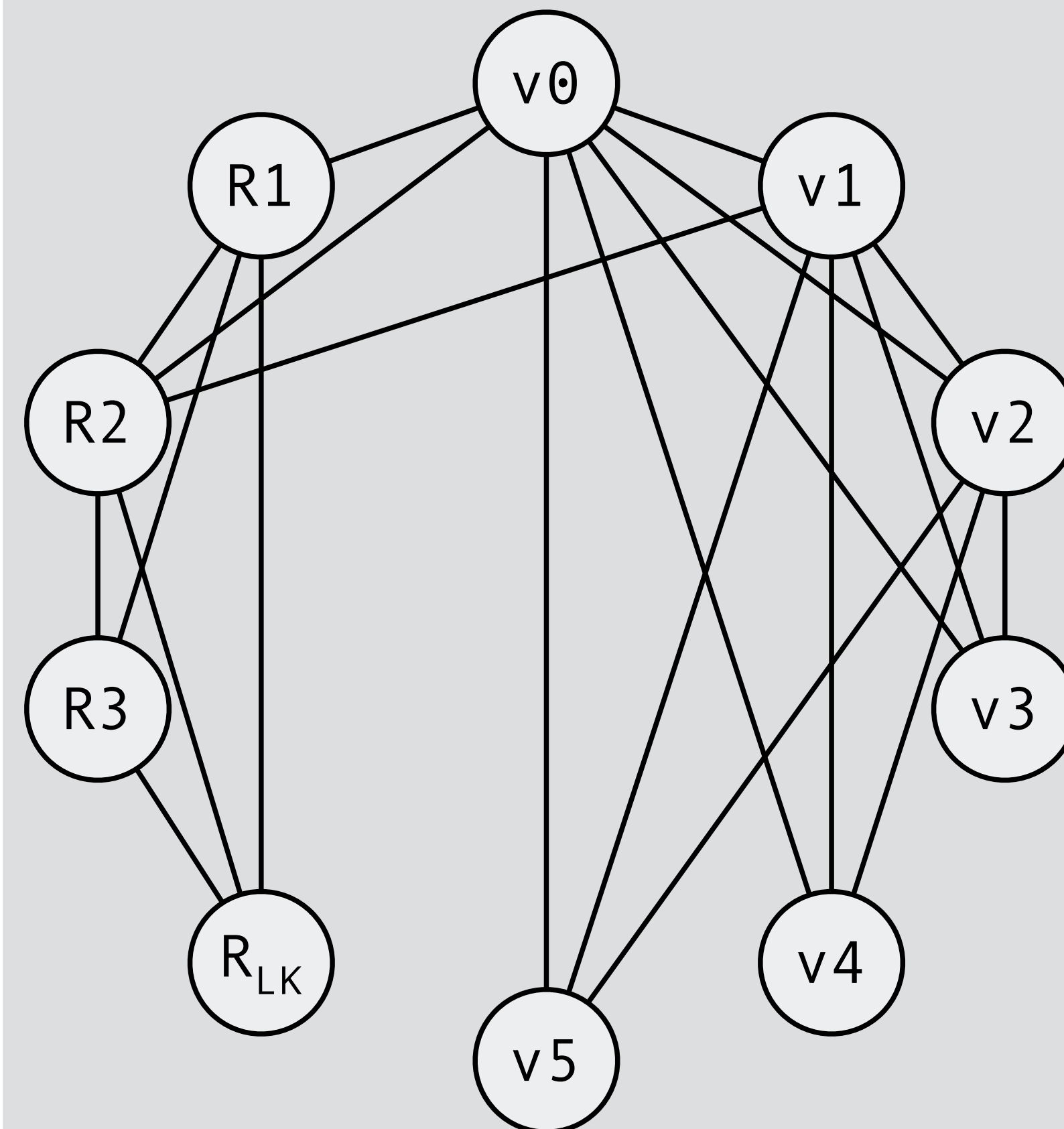
```
gcd:
  v0 ← RLK
  v1 ← R1
  v2 ← R2
loop:
  v3 ← done
  if v2=0 goto v3
  v4 ← v2
  v2 ← v1 % v2
  v1 ← v4
  v5 ← loop
  goto v5
done:
  R1 ← v1
  goto v0
```

Liveness

{in}{out}

$\{R_1, R_2, R_{LK}\} \{R_1, R_2, v_0\}$
 $\{R_1, R_2, v_0\} \{R_2, v_0, v_1\}$
 $\{R_2, v_0, v_1\} \{v_0 - v_2\}$
 $\{v_0 - v_2\} \{v_0 - v_3\}$
 $\{v_0 - v_3\} \{v_0 - v_2\}$
 $\{v_0 - v_2\} \{v_0 - v_2, v_4\}$
 $\{v_0 - v_2, v_4\} \{v_0 - v_2, v_4\}$
 $\{v_0 - v_2, v_4\} \{v_0 - v_2\}$
 $\{v_0 - v_2\} \{v_0 - v_2, v_5\}$
 $\{v_0 - v_2, v_5\} \{v_0 - v_2\}$
 $\{v_0, v_1\} \{R_1, v_0\}$
 $\{R_1, v_0\} \{R_1\}$

Interference graph

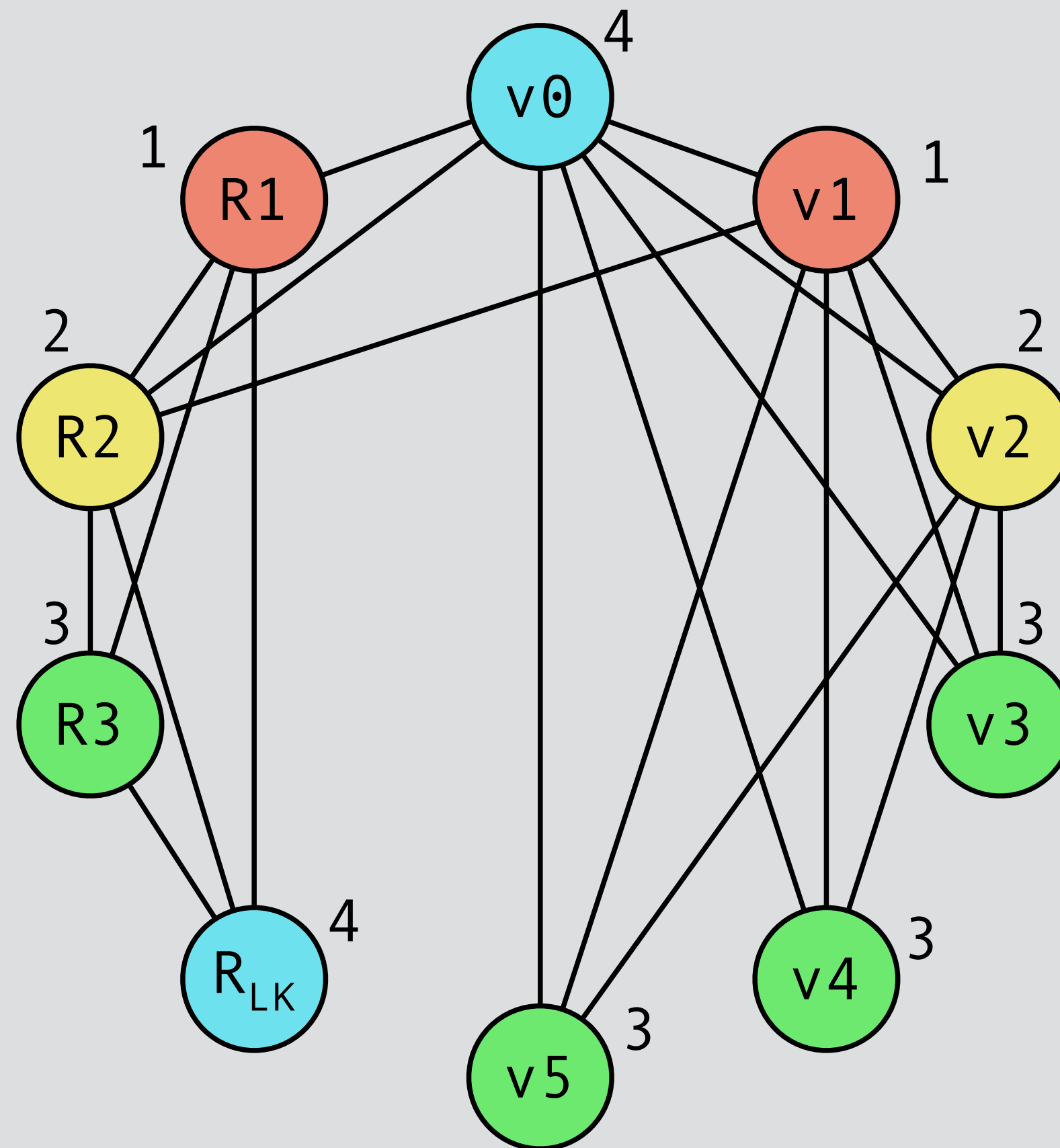


Coloring example

Original prog.

```
gcd:
  v0 ← RLK
  v1 ← R1
  v2 ← R2
loop:
  v3 ← done
  if v2=0 goto v3
  v4 ← v2
  v2 ← v1 % v2
  v1 ← v4
  v5 ← loop
  goto v5
done:
  R1 ← v1
  goto v0
```

Colored interference graph



Rewritten prog.

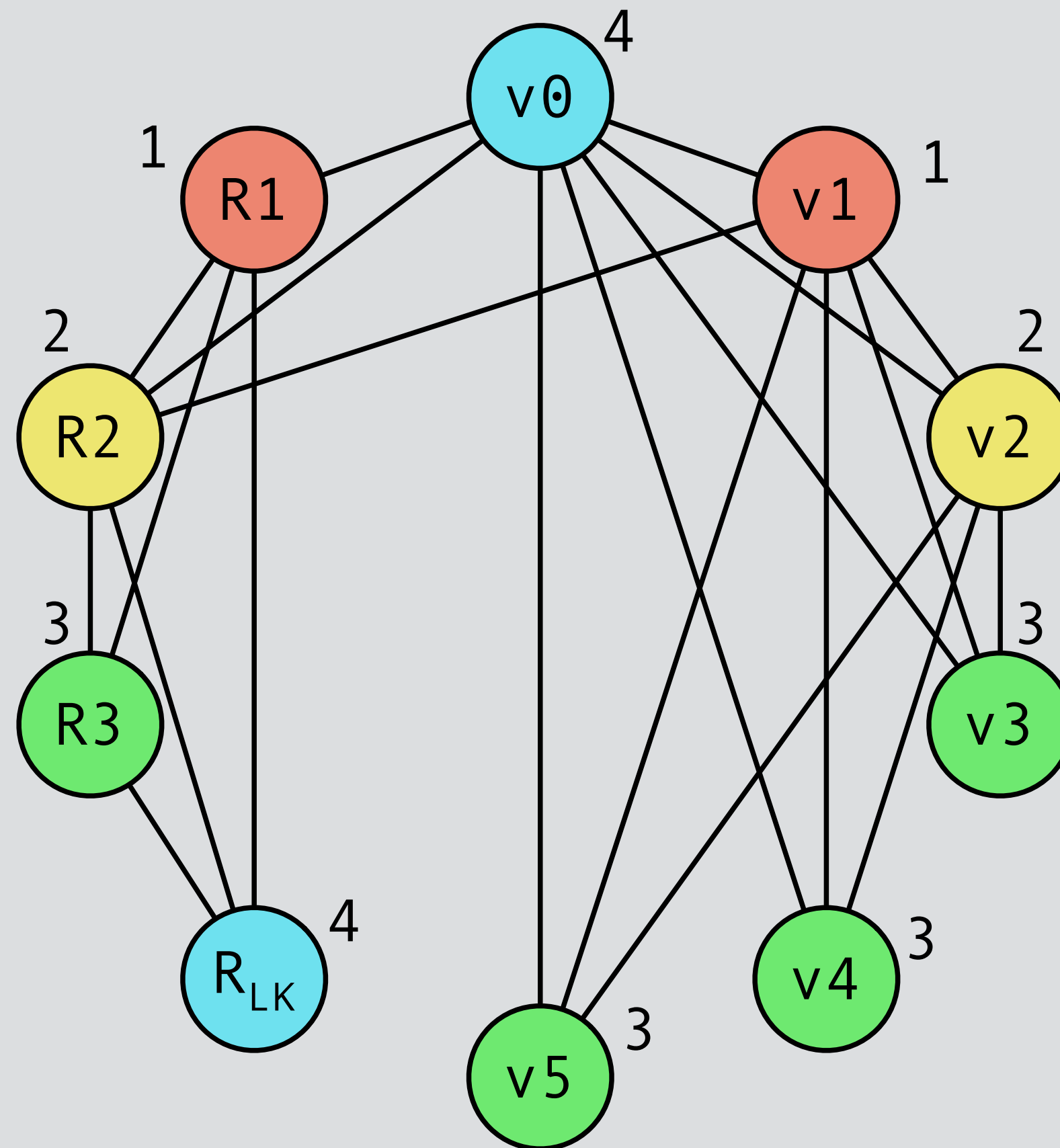
```
gcd:
  RLK ← RLK
  R1 ← R1
  R2 ← R2
loop:
  R3 ← done
  if R2=0 goto R3
  R3 ← R2
  R2 ← R1 % R2
  R1 ← R3
  R3 ← loop
  goto R3
done:
  R1 ← R1
  goto RLK
```

Coloring example

Original prog.

```
gcd:
  v0 ← RLK
  v1 ← R1
  v2 ← R2
loop:
  v3 ← done
  if v2=0 goto v3
  v4 ← v2
  v2 ← v1 % v2
  v1 ← v4
  v5 ← loop
  goto v5
done:
  R1 ← v1
  goto v0
```

Colored interference graph



Rewritten prog.

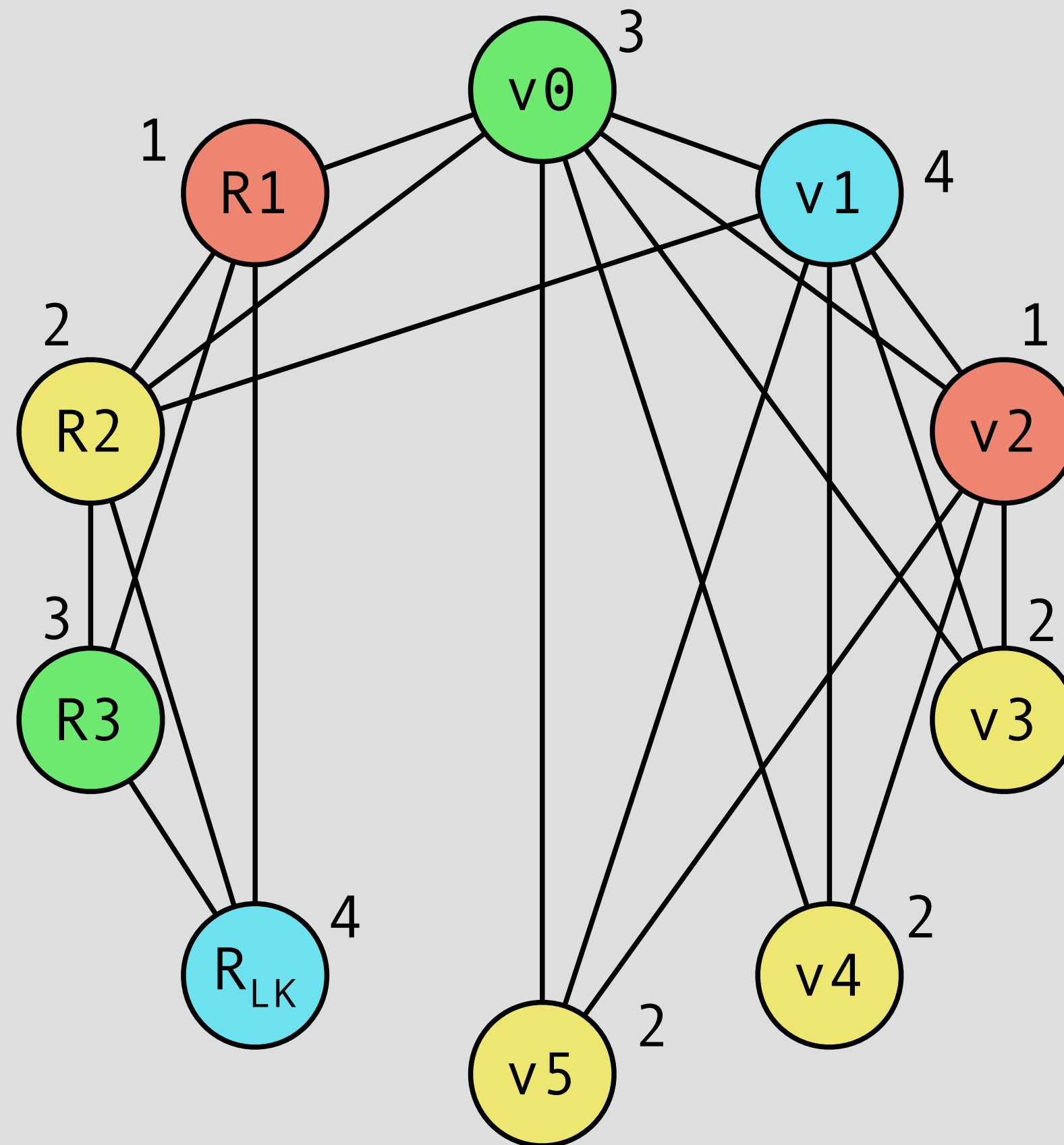
```
gcd:
RLK ← RLK
R1 ← R1
R2 ← R2
loop:
  R3 ← done
  if R2=0 goto R3
  R3 ← R2
  R2 ← R1 % R2
  R1 ← R3
  R3 ← loop
  goto R3
done:
R1 ← R1
  goto RLK
```

Coloring example (2)

Original prog.

```
gcd:
  v0 ← RLK
  v1 ← R1
  v2 ← R2
loop:
  v3 ← done
  if v2=0 goto v3
  v4 ← v2
  v2 ← v1 % v2
  v1 ← v4
  v5 ← loop
  goto v5
done:
  R1 ← v1
  goto v0
```

Colored interference graph



Rewritten prog.

```
gcd:
  R3 ← RLK
  RLK ← R1
  R1 ← R2
loop:
  R2 ← done
  if R1=0 goto R2
  R2 ← R1
  R1 ← RLK % R1
  RLK ← R2
  R2 ← loop
  goto R2
done:
  R1 ← RLK
  goto R3
```

This second coloring is also correct, but produces worse code!

Coloring by simplification

Coloring by simplification is a heuristic technique to color a graph with K colors:

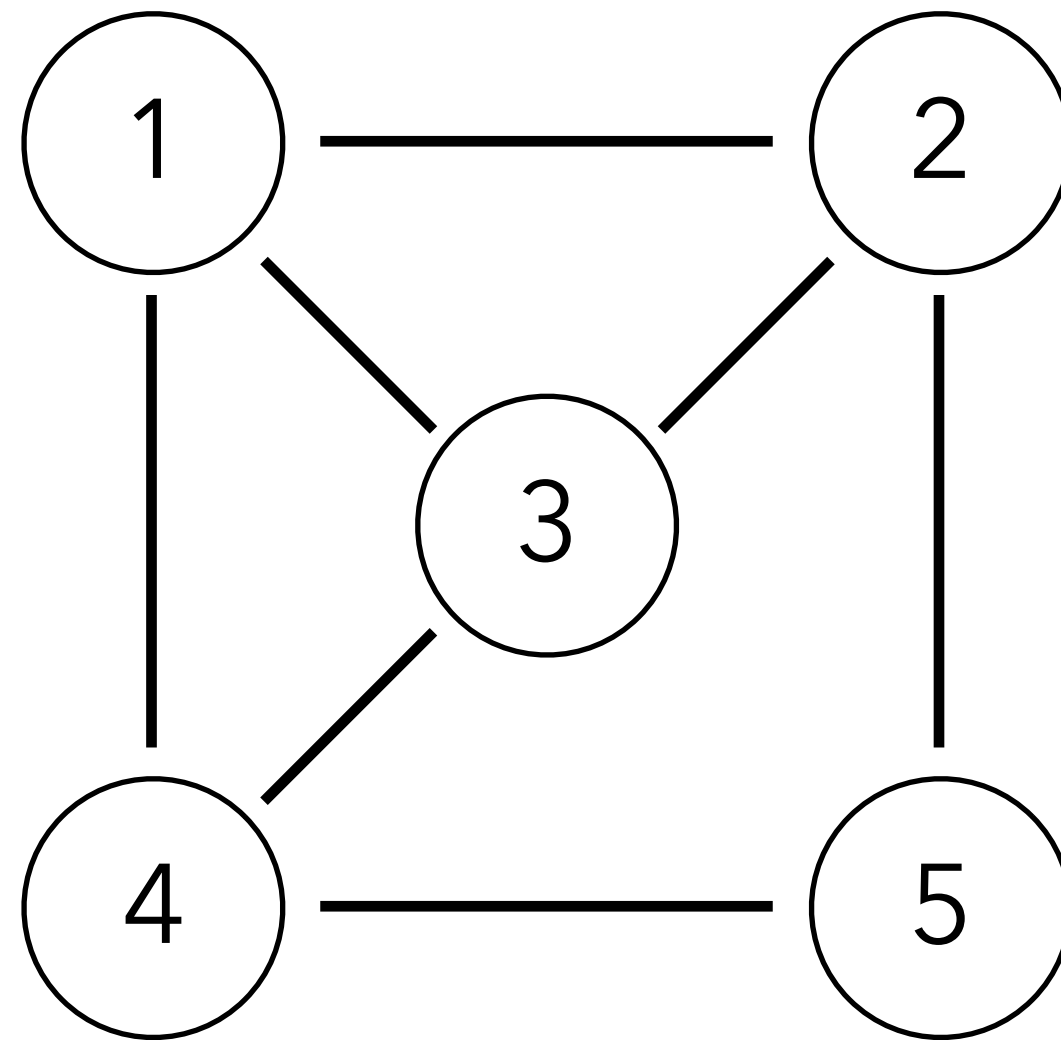
1. find a node n with less than K neighbors,
2. remove it from the graph,
3. recursively color the simplified graph,
4. color n with any color not used by its neighbors.

What if there is no node with less than K neighbors?

- a K -coloring might not exist,
- but simplification is attempted nevertheless.

Coloring by simplification

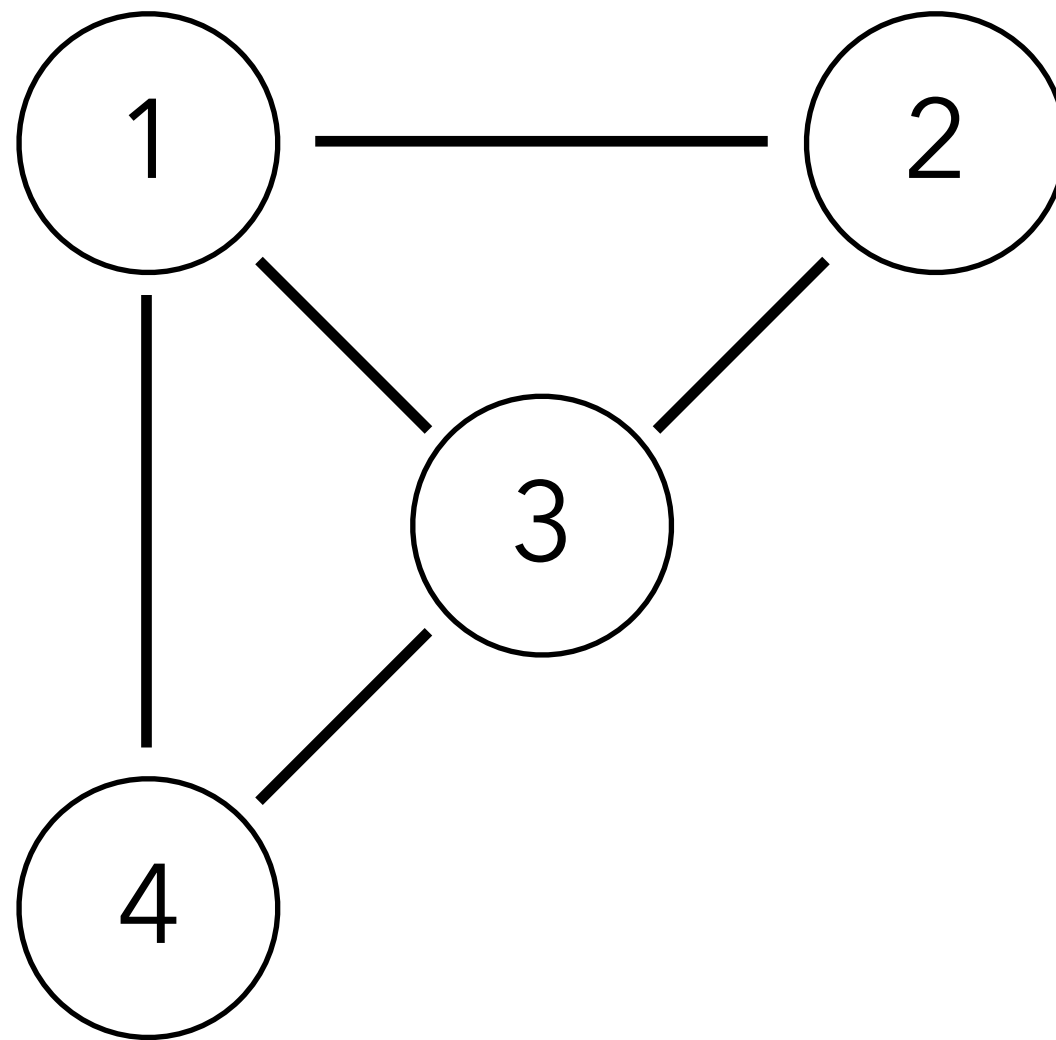
Number of available colors (K): 3



Stack of removed nodes:

Coloring by simplification

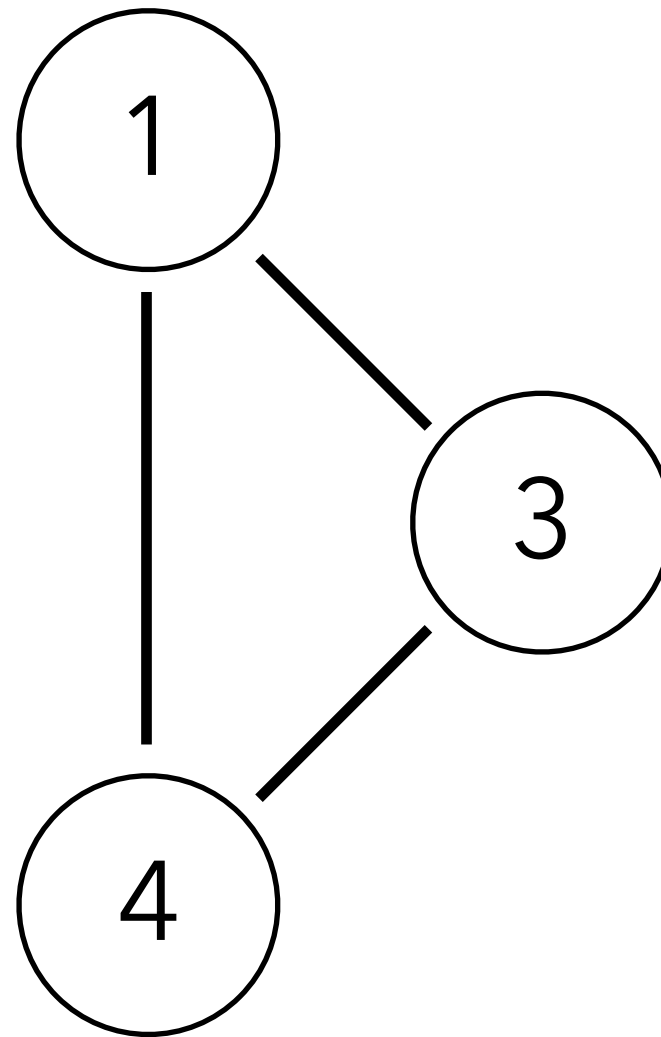
Number of available colors (K): 3



Stack of removed nodes: 5

Coloring by simplification

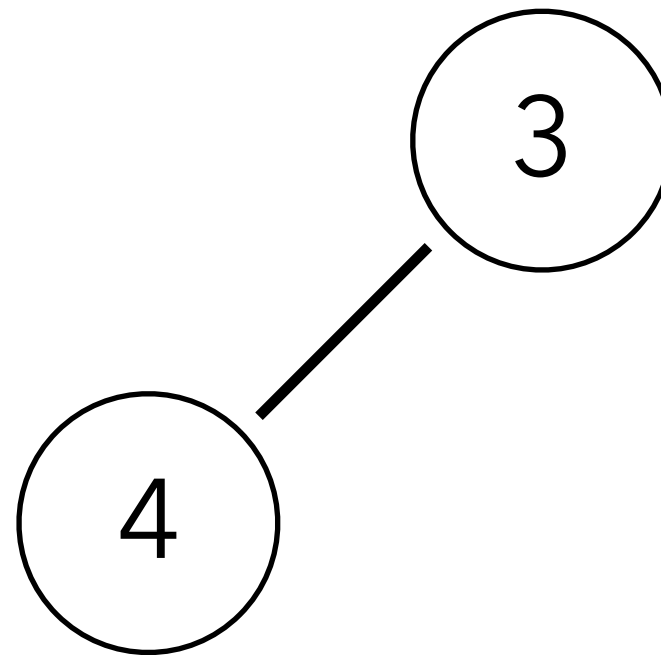
Number of available colors (K): 3



Stack of removed nodes: 5 2

Coloring by simplification

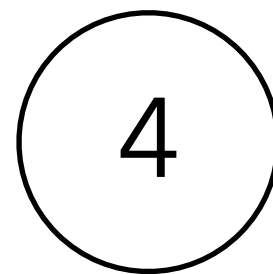
Number of available colors (K): 3



Stack of removed nodes: 5 2 1

Coloring by simplification

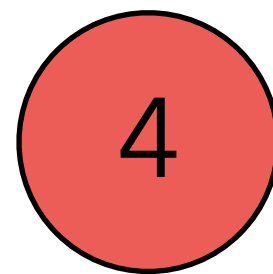
Number of available colors (K): 3



Stack of removed nodes: 5 2 1 3

Coloring by simplification

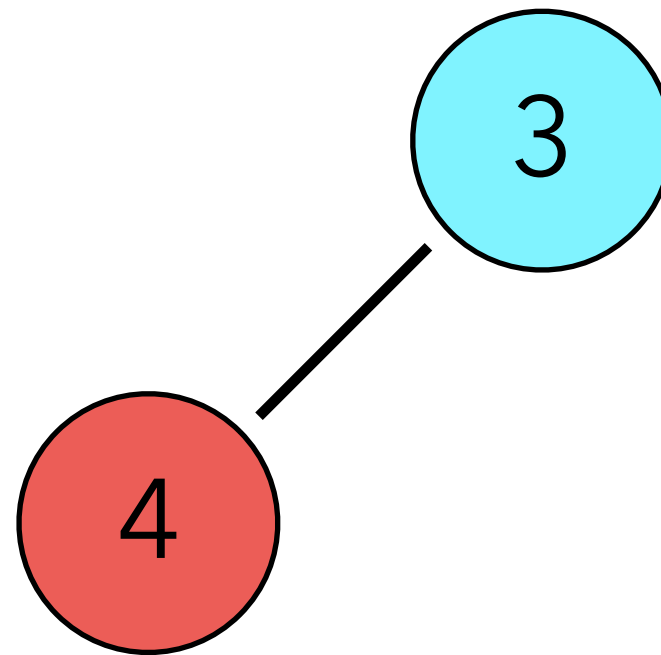
Number of available colors (K): 3



Stack of removed nodes: 5 2 1 3

Coloring by simplification

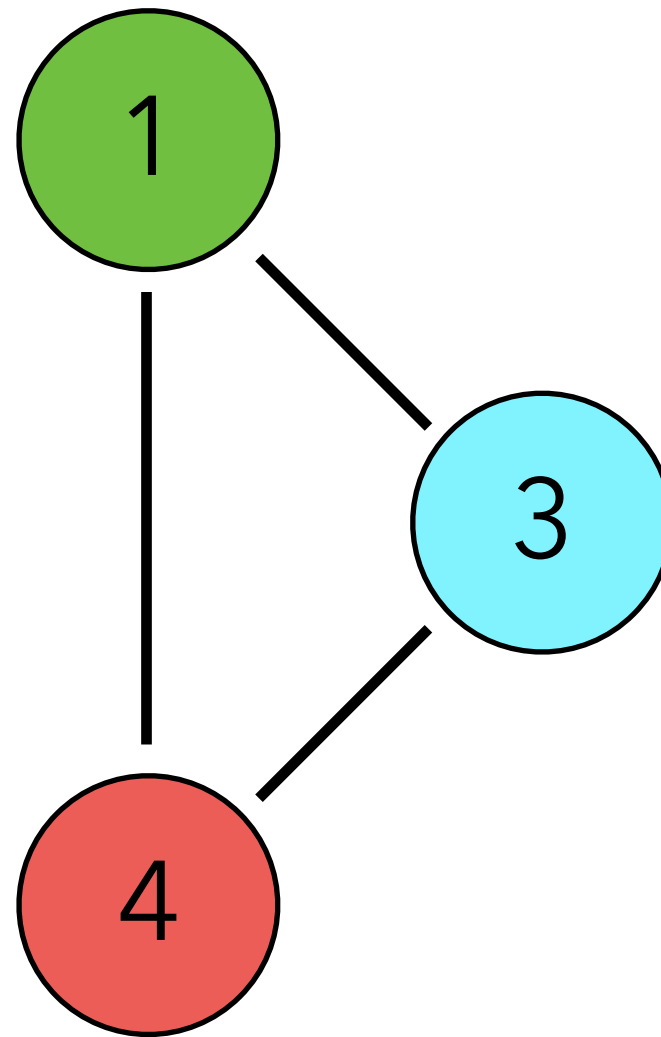
Number of available colors (K): 3



Stack of removed nodes: 5 2 1

Coloring by simplification

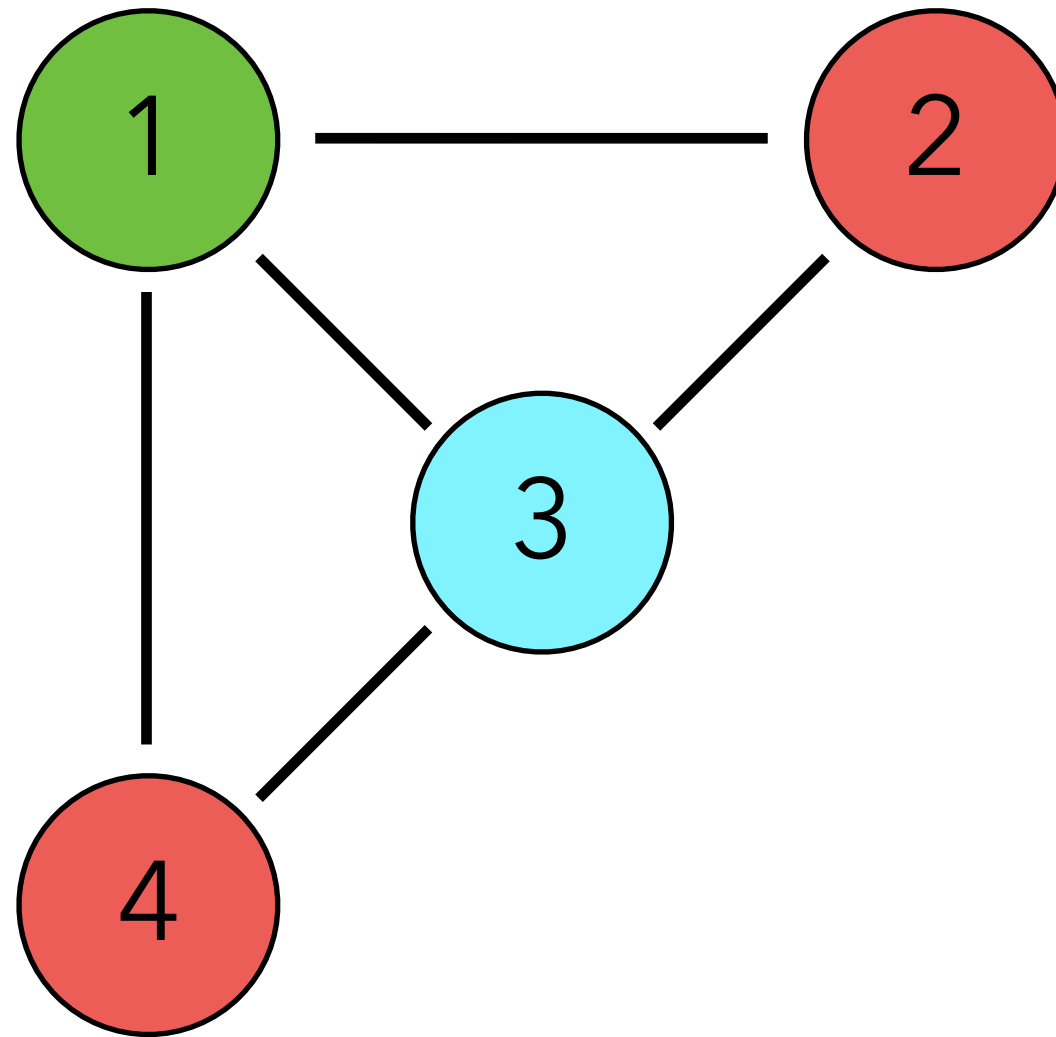
Number of available colors (K): 3



Stack of removed nodes: 5 2

Coloring by simplification

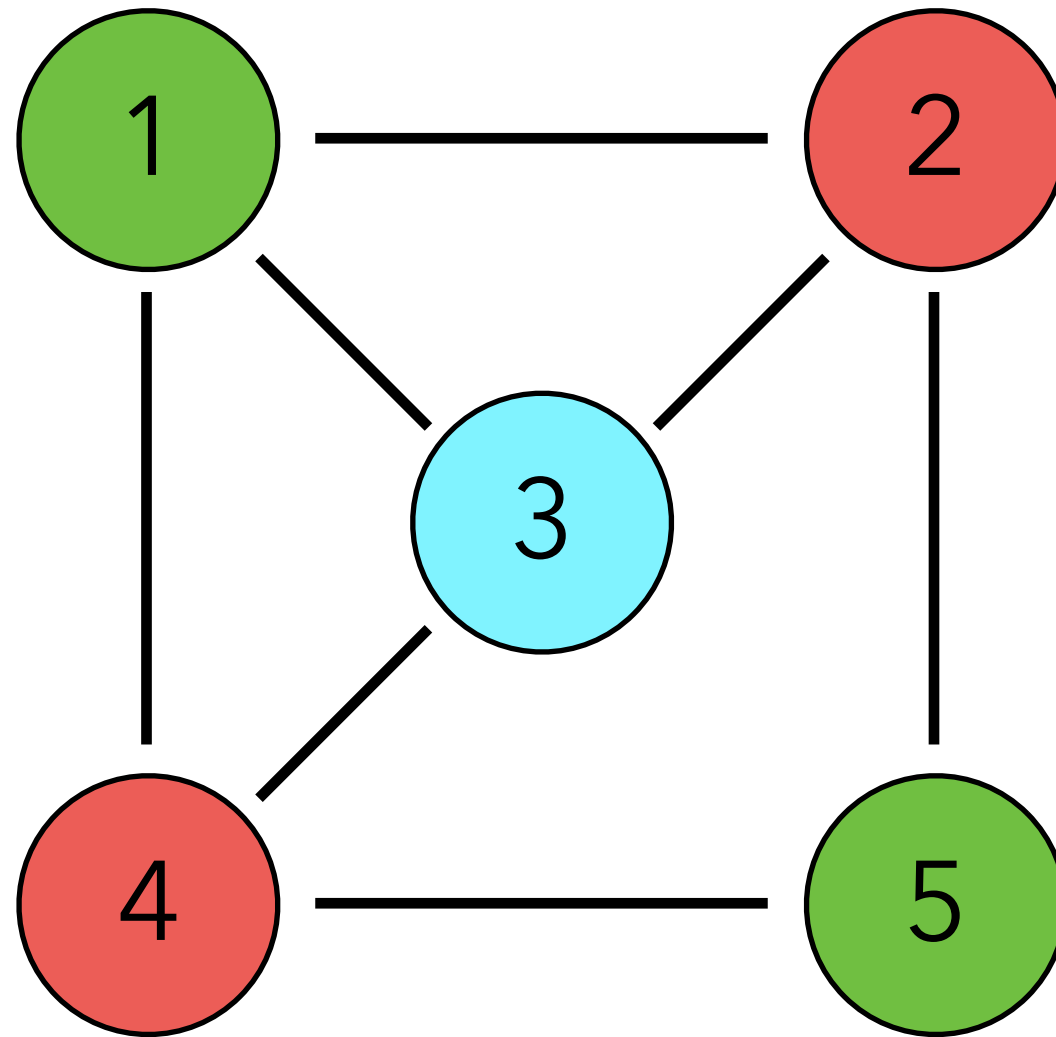
Number of available colors (K): 3



Stack of removed nodes: 5

Coloring by simplification

Number of available colors (K): 3



Stack of removed nodes:

Spilling

(Optimistic) spilling

What if all nodes have K or more neighbors during simplification?

A node n must be chosen to be **spilled** and its value stored in memory instead of in a register:

- remove its node from the graph (assuming no interference between spilled value and other values),
- recursively color the simplified graph as usual.

Once recursive coloring is done, two cases:

1. by chance, the neighbors of n do not use all the possible colors, n is not spilled,
2. otherwise, n is really spilled.

Spill costs

Which node should be spilled? Ideally one:

- whose value is not frequently used, and/or
- that interferes with many other nodes.

For that, compute the spill cost of a node n as:

$$\text{cost}(n) = (\text{rw}_0(n) + 10 \text{rw}_1(n) + \dots + 10^k \text{rw}_k(n)) / \text{degree}(n)$$

where:

- $\text{rw}_i(n)$ is the number of times the value of n is read or written in a loop of depth i ,
- $\text{degree}(n)$ is the number of edges adjacent to n in the interference graph.

Then spill the node with lowest cost.

Spilling of pre-colored nodes

The interference graph contains nodes corresponding to the physical registers of the machine:

- they are said to be **pre-colored**, as their color is given by the machine register they represent,
- they should never be simplified, as they cannot be spilled (they are physical registers!).

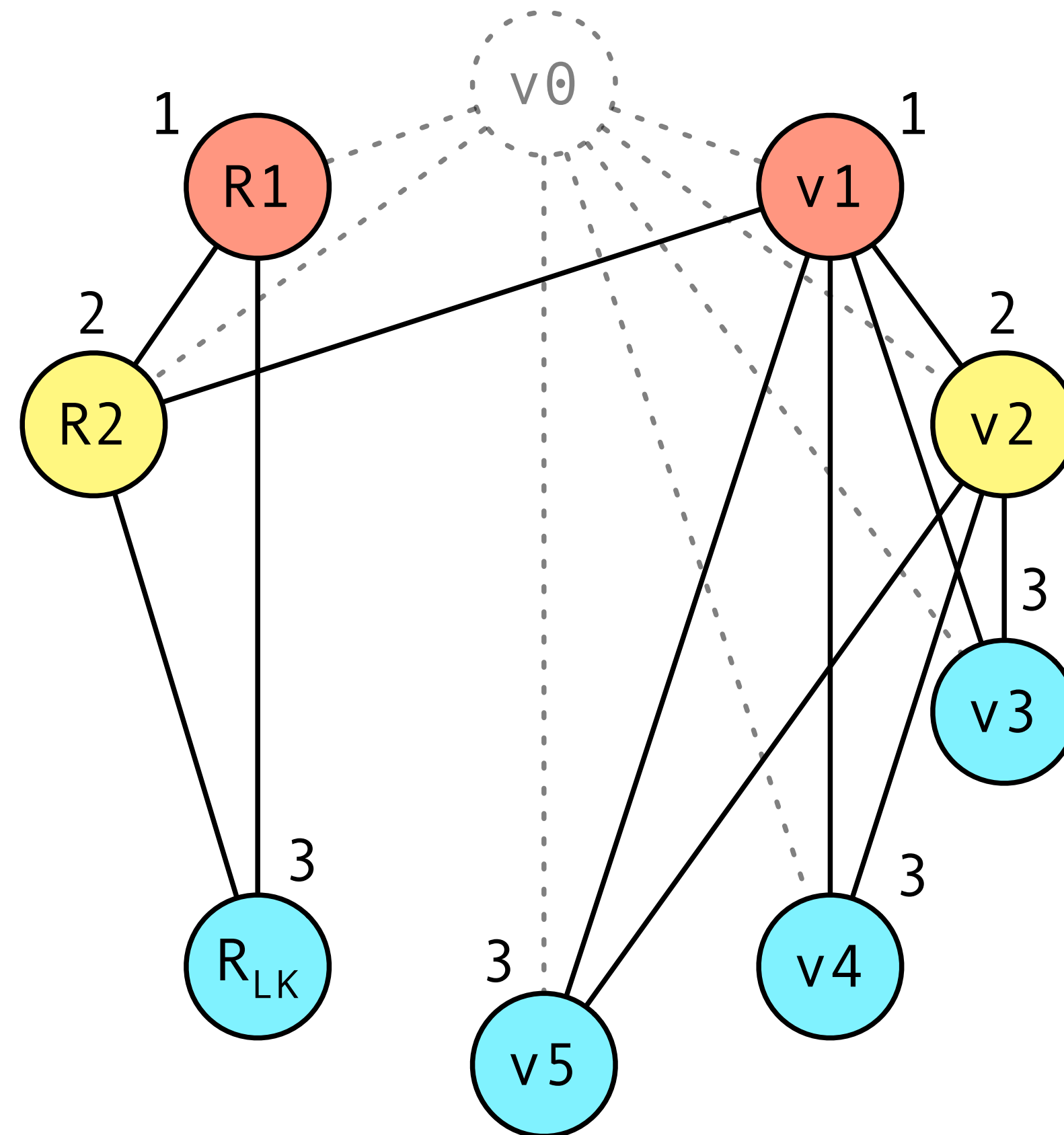
Spilling example: costs

```
gcd:
  v0 ← RLK
  v1 ← R1
  v2 ← R2
loop:
  v3 ← done
  if v2=0 goto v3
  v4 ← v2
  v2 ← v1 % v2
  v1 ← v4
  v5 ← loop
  goto v5
done:
  R1 ← v1
  goto v0
```

node	rw ₀	rw ₁	deg.	cost
v ₀	2	0	7	0.29
v ₁	2	2	6	3.67
v ₂	1	4	6	6.83
v ₃	0	2	3	6.67
v ₄	0	2	3	6.67
v ₅	0	2	3	6.67

$$\text{cost} = (\text{rw}_0 + 10 \text{rw}_1) / \text{degree}$$

Spilling example



Consequences of spilling

After spilling, rewrite the program to:

- insert code just before the spilled value is read, to fetch it from memory,
- insert code just after the spilled value is written, to write it back to memory.

But: spilling code introduces new virtual registers, so register allocation must be redone!

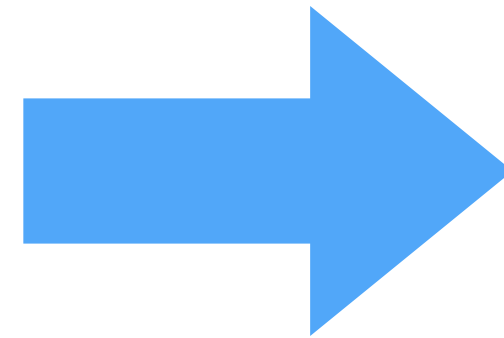
In practice, 1-2 iterations are enough in almost all cases.

Spilling code integration

Original program

```
gcd:
  v0 ← RLK
  v1 ← R1
  v2 ← R2
loop:
  v3 ← done
  if v2 = 0 goto v3
  v4 ← v2
  v2 ← v1 % v2
  v1 ← v4
  v5 ← loop
  goto v5
done:
  R1 ← v1
  goto v0
```

spilling
of v₀



Rewritten program

```
gcd:
  v6 ← RLK
  push v6
  v1 ← R1
  v2 ← R2
loop:
  v3 ← done
  if v2 = 0 goto v3
  v4 ← v2
  v2 ← v1 % v2
  v1 ← v4
  v5 ← loop
  goto v5
done:
  R1 ← v1
  pop v7
  goto v7
```


Coalescing

Coalescing

If v_1 and v_2 do not interfere, a move instruction of the form

$$v_1 \leftarrow v_2$$

can always be removed by replacing v_1 and v_2 by a new virtual register $v_{1\&2}$.

This is called **coalescing**, as the nodes of v_1 and v_2 in the interference graph coalesce into a single node.

